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*Focussed on characterization of crack tip fields*

# A numerical study of non-linear crack tip parameters

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**ABSTRACT.** Crack closure concept has been widely used to explain different issues of fatigue crack propagation. However, different authors have questioned the relevance of crack closure and have proposed alternative concepts. The main objective here is to check the effectiveness of crack closure concept by linking the contact of crack flanks with non-linear crack tip parameters. Accordingly, 3D-FE numerical models with and without contact were developed for a wide range of loading scenarios and the crack tip parameters usually linked to fatigue crack growth, namely range of cyclic plastic strain, crack tip opening displacement, size of reversed plastic zone and total plastic dissipation per cycle, were investigated. It was demonstrated that: i) LEFM concepts are applicable to the problem under study; ii) the crack closure phenomenon has a great influence on crack tip parameters decreasing their values; iii) the $\Delta K_{eff}$ concept is able to explain the variations of crack tip parameters produced by the contact of crack flanks; iv) the analysis of remote compliance is the best numerical parameter to quantify the crack opening level; v) without contact there is no effect of stress ratio on crack tip parameters. Therefore it is proved that the crack closure concept is valid.

**KEYWORDS.** Fatigue crack propagation; Plasticity induced crack closure; Non-linear crack tip parameters; $\Delta K_{eff}$

## INTRODUCTION

Modern design methodologies consider that inherent defects are always present in components. Fatigue life is therefore defined as the number of load cycles required to propagate these defects up to a critical size. Engineering analysis of fatigue crack propagation is usually performed by relating the crack advance per unit cycle, da/dN, to the stress intensity factor range, ΔK. Initially it was surprising that this linear-elastic parameter could successfully describe the rate of plastic processes at the crack tip. Rice [1] showed that the small-scale cyclic plasticity at the crack tip is, indeed, controlled by ΔK. According to Paris law, da/dN is uniquely determined by one loading parameter, the stress intensity factor range, $\Delta K$. However, the large amount of work developed showed that other parameters influence da/dN, like stress ratio or load history. Christensen [2] proposed the concept of fracture surface interaction leading to a decrease of stress intensity at the crack tip and to an increase of fatigue life. Elber [3] discussed the concept in terms of fracture mechanics parameters, promoting a strong research effort into the mechanisms and phenomena associated with fatigue crack closure. Ritchie *et al* [4] and Suresh [5,6] identified the main closure mechanisms, which are plasticity induced crack closure (PICC), oxide-induced crack closure and roughness induced crack closure. According to Elber's understanding of crack closure, as the crack propagates due to cyclic loading, a residual plastic wake is formed. The deformed material acts as a wedge behind the crack tip and the contact of fracture surfaces is forced by the elastically deformed material. Crack closure concept seemed to be able to explain the influence of mean stress in both regimes I and II of crack propagation [7], the transient crack growth behaviour following overloads [8], the growth rate of short cracks [9] and the effect of thickness [10, 11], among other aspects. This success in explaining different issues of fatigue crack propagation has been used to validate the crack closure concept. Pippan and Grosinger [12] demonstrated that crack closure is not only important under small scale yielding conditions, it is also essential in the regions of low cycle fatigue. The effect of specimen geometry on crack closure has been accounted for using the T-stress concept [13]. Complementary concepts have been proposed by different authors. Dai and Li [14] considered that the plastic deformation modifies the elastic stress field and defined a plasticity-corrected K to account for the effect of plasticity. This $K_{pc}$ was proposed as a new mechanical driving force parameter for predicting FCG rate, able to explain important phenomena associated with the plastic zone around a fatigue crack tip, such as the effects of load ratio R, single overload and the FCG behavior under cyclic compression. Ranc *et al.* [15] quantified the effect of heterogeneous temperature on stress intensity factor. The energy dissipated in the cyclic plastic zone ahead of crack tip produces thermal expansion of the material which affects the stress field. The stress intensity factor has to be corrected by a negative value which reduces the crack driving force. Pokluda [16] states that the effective stress field at the crack tip is a superposition of remote and local SIFs. The internal stresses created by dislocation configurations and secondary phases are to be considered as an important additional factor affecting the crack propagation rate in fatigue. Christopher *et al.* [17, 18] proposed a novel mathematical model of the stresses around the tip of a fatigue crack, which considers the effects of wake contact and compatibility-induced stresses at the elastic–plastic boundary. Four parameters were considered to characterize the stress field: an opening mode stress intensity factor $K_F$, the shear stress intensity factor $K_S$, the retardation stress intensity factor $K_R$, and the T-stress. $K_R$ characterizes the effect of crack tip shielding arising due to plasticity both at the crack tip and in the wake.

However, several questions have been raised questioning the crack closure concept, therefore the importance and even the existence of crack closure effect have been questioned by different authors. Donald and Paris [19] and Kujawski [20] introduced the concept of partial crack closure, which indicates that the contact of crack flanks at some distance from crack tip has a relatively low effect on FCGR. Some researchers suggested that closure can only occur under plane stress [21], while others believe that it may not occur at all. Since 1993 Sadananda and Vasudevan [22-24] have advocated that because the closure occurs behind the crack tip, it has a rather limited effect on the damage process, which takes place at the 'process zone' in front of the crack. According to these researchers the approaches to fatigue behavior based on crack closure (i.e. on what happens behind the crack tip) should be replaced by approaches based on what happens ahead of the crack tip. They argued that closure effects on FCG behavior have been greatly exaggerated, and suggested that the fatigue crack propagation rate is controlled by a two parameter driving force, which is a function of the maximum stress intensity factor, $K_{max}$, and total stress intensity factor range, $\Delta K$. These two parameters account for both the applied load and the residual stress contributions. Kujawski [25] proposed a new driving force parameter for crack growth: $\Delta K_{effK}=(K_{max}\Delta K^{+})^{0.5}$, being $\Delta K^{+}$ the positive part of $\Delta K$. He found that without using the crack closure concept, it is possible to explain the stress ratio effect, even better than using this concept. However, Noroozi *et al.* [26, 27] pointed out that these models are strictly empirical and cannot explain the influence of the compressive part of the load history on fatigue crack growth. They formulated a unified two-parameter model to correlate $K_{max}$ and $\Delta K$ with the actual elastic-plastic crack tip stress–strain field. In their investigation, the difference in the stress–strain concentration at the crack tip associated with the compressive part of the loading cycle was taken into account.

Clearly there is no general agreement among researchers regarding the significance of closure concept on fatigue crack behavior. The contact of crack flanks is accepted by all, because it was observed numerically and using experimental techniques, namely, digital image correlation [28], x-ray diffraction [28,29], potential drop [30,31] and SEM [31]. The great disagreement is about the effect of this contact on fatigue crack growth. In fact, the direct link between crack closure and crack tip fields has not been totally exploited. This might be due to experimental difficulties in measuring quantitative

strain/stress fields near a fatigue crack tip [30]. Anyway, different studies may be found in literature [32-34]. The link between crack closure and non-linear crack tip parameters is however rare. Further work is therefore necessary to quantify the effect of the contact of crack flanks on the process zone where the propagation effectively happens.
The main objective of this paper is therefore to check the effectiveness of crack closure concept by linking the contact of crack flanks to the non-linear crack tip parameters. A M(T) specimen made of 6016-T4 aluminium alloy was modeled and submitted to different load scenarios using the finite element method. The numerical tests were done with and without contact of crack flanks. The numerical approaches are very interesting for the elimination of contact in order to study its effect, however no studies were reported in literature by the authors.

## NON-LINEAR CRACK TIP PARAMETERS (NLP)

Fig. 1 shows the four different zones that can be identified ahead of a fatigue crack tip [35]. In the elastic zone (regions I and II), which is far ahead of crack tip, the material is deformed in purely elastic manner. The stress intensity factor controls the magnitude of stress and strain fields in region II. Region III is known as monotonic plastic zone. Plastic deformation occurs during monotonic loading and after that elastic loading-unloading is taking place. In region IV, close to fatigue crack-tip, known as reverse/cyclic plastic zone, hysteresis loop occurs. The small scale yielding hypothesis justifies the use of ΔK as the crack driving force. However, it provides no information about the physical phenomena happening during crack propagation, namely in the reversed plastic zone. It is widely accepted by the scientific community that crack advance in metals is mainly determined by the damage of a highly localized volume immediately ahead of the crack tip, called the process zone. A literature review was made to identify the crack tip parameters that may be expected to control crack tip progression due to cyclic loading.

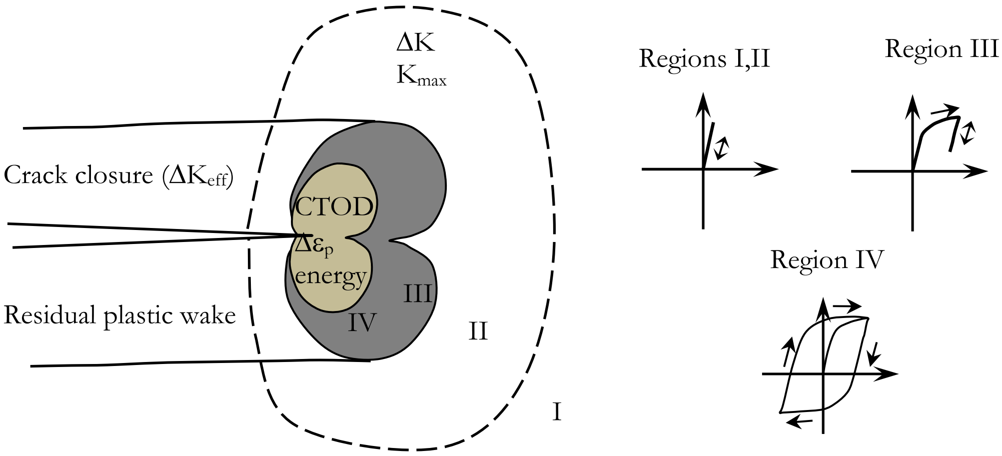


Figure 1: Schematic diagram of crack tip zones, parameters and stress-strain response.

Pokluda [16] stated that the crack driving force in fatigue is directly related to the range of cyclic plastic strain. The crack tip opening displacement (CTOD or COD) is another main crack tip parameter. Note that the COD is equal to ΔCOD since the crack re-sharpens during unloading [36]. Pelloux [37], using microfractography, showed that the concept of COD allowed the prediction of fatigue striations spacing and therefore the crack growth rate. Nicholls [38] assumed a polynomial relation between crack growth rate and CTOD:

$$\frac{da}{dN} = b(CTOD)^{1/p} \tag{1}$$

where b and p are constants. Tvergaard [39] and Pippan and Grosinger [12] indicated a linear relation between da/dN and CTOD for very ductile materials:

$$\frac{da}{dN} = c \times CTOD \tag{2}$$

being $c$ a constant. In numerical studies the CTOD is usually defined as the distance between two points found by intersecting the finite element model with two (+45º and -45º) lines originated from the crack tip. The size of reversed plastic zone has also been considered a main parameter of crack growth [40, 41]. Ould Chick *et al.* [42] showed that da/dN has a linear variation with the square of the cyclic plastic zone size ($r_{pc}^2$):

$$\frac{da}{dN} = A(r_{pc})^2 \qquad (3)$$

where A depends on the yield stress. Other authors suggested that the total plastic dissipation per cycle occurring in the reversed plastic zone is a driving force for fatigue crack growth in ductile solids, and can be closely correlated with fatigue crack growth rates [43, 44]. Dissipated energy approaches to fatigue crack growth prediction have since been the subject of numerous analytical [45, 46] and experimental [47, 48] investigations.

## NUMERICAL MODEL

A Middle-Tension specimen was considered to predict the crack opening level, having W=60 mm and a straight crack with an initial size $a_0$ of 5 mm ($a_0$/W=0.083). A small thickness was considered (t=0.1 mm) to simulate the plane stress state. Two materials were considered in this research: the 6016-T4 aluminium alloy and a High Strength Steel (DP600). Since PICC is a plastic deformation based phenomenon, the hardening behaviour of the material was carefully modelled. The hardening behaviour of this alloy was represented using an isotropic hardening model described by a Voce type equation, combined with a non-linear kinematic hardening model described by a saturation law. Table 1 indicates the load parameters defined in the different sets of constant amplitude tests considered for 6016-T4 aluminium alloy and DP600 steel, respectively. Sets with constant $K_{min}$, $K_{max}$, ΔK and R were studied, as can be seen.

| Set 1 ($K_{min}$=0) | | Set 2 ($K_{max}$=6.4) | | Set 3 ($K_{max}$=2.2) | | Set 4 ($K_{max}$=4.6) | |
|---|---|---|---|---|---|---|---|
| ΔK | R | ΔK | R | ΔK | R | ΔK | R |
| 2.9 | 0 | 3.8 | 0.43 | 2.2 | 0 | 2.3 | 0.5 |
| 3.8 | 0 | 5.7 | 0.14 | 4.4 | -1 | 4.6 | 0.0 |
| 4.8 | 0 | 7.7 | -0.14 | 6.6 | -2 | 6.8 | -0.5 |
| 6.7 | 0 | 9.6 | -0.43 | 8.9 | -3 | 9.1 | -1 |
| 8.6 | 0 | 11.5 | -0.71 | 11.0 | -4 | 12.5 | -1.75 |
| 9.6 | 0 | 13.4 | -1.00 | 13.6 | -5 | 13.3 | -2 |
| 10.5 | 0 | 15.3 | -1.29 | 15.9 | -6 | 14.8 | -2.25 |
| Set 5 (R=0.2) | | Set 6 (ΔK=4.8) | | Set 7 (ΔK=6.7) | | Set 8 ($K_{max}$=9.1) | |
| ΔK | R | ΔK | R | ΔK | R | ΔK | R |
| 3.1 | 0.2 | 4.8 | -2 | 6.7 | -2 | 1.4 | 0.88 |
| 3.8 | 0.2 | 4.8 | -1 | 6.7 | -1 | 2.5 | 0.75 |
| 4.6 | 0.2 | 4.8 | -0.5 | 6.7 | -0.5 | 4.6 | 0.5 |
| 5.4 | 0.2 | 4.8 | 0 | 6.7 | 0 | 6.9 | 0.25 |
| 6.1 | 0.2 | 4.8 | 0.25 | 6.7 | 0.25 | 9.1 | 0 |
| 6.9 | 0.2 | 4.8 | 0.5 | 6.7 | 0.5 | 11.3 | -0.25 |

Table 1: Loading parameters for 6016-T4 aluminium alloy ([ΔK, $K_{max}$, $K_{min}$]=MPa.m$^{1/2}$)

The finite element mesh considered was refined near the crack tip and enlarged at relatively remote positions. Square elements with 8×8 $\mu m^2$ were defined in the refined region, while only one layer of elements was considered along the thickness. Crack propagation was simulated by successive debonding of nodes at the minimum load. Each crack increment corresponded to one finite element and two load cycles were applied between increments. In each cycle, the crack propagates uniformly over the thickness by releasing both current crack front nodes. The opening load, $F_{op}$, necessary for the determination of the closure level was obtained considering the contact status of the first node behind the current crack tip. The numerical simulations were performed with the Three-Dimensional Elastic-plastic Finite Element program (DD3IMP), originally developed to simulate deep drawing. Further details of this numerical procedure may be found in literature [49].

The analysis of the effect of contact flanks was developed comparing the crack tip parameters obtained with and without contact. For each load condition, the crack was submitted to 160 crack increments and 320 load cycles, which corresponds to a global crack increment $\Delta a$=160×8μm=1.280 mm. This is enough to stabilize the crack opening values. After that, 30 load cycles were applied without crack propagation. This procedure was done with and without the symmetry plane used to simulate the contact of crack flanks. Three non-linear crack tip parameters were measured at the end of this procedure: the crack tip opening displacement (COD), the range of plastic strain ($\Delta\varepsilon_{p,yy}$), and the energy dissipated per cycle. These last two quantities were measured at the Gauss point immediately ahead of the last crack tip position, and in the last load cycle applied. The energy is the area of the last stress-strain loop. Note that da/dN is usually correlated with the total energy dissipated ahead of crack tip. Anyway the energy at the point immediately ahead of the crack tip is related with the total energy. The size of cyclic plastic zone was determined from the analysis of equivalent plastic strain ahead of crack tip. The increase of plastic deformation with the decrease of load, down to its minimum value, indicates the occurrence of reversed plasticity. The COD was assumed to be the vertical displacement of the node behind crack tip at maximum load. The same approach was used by Ellyin and Wu [50] to quantify the COD.

## NUMERICAL RESULTS

### *Validity of LEFM*

The linear elastic fracture mechanics (LEFM) assume that the crack tip process zone is controlled by the elastic field around it, i.e., that the K concept is valid. Small-scale plasticity must however exist, otherwise K will not be the controlling parameter. The validity of LEFM was checked here, verifying the relation between $\Delta K$ and the non-linear crack tip parameters. Fig. 2a plots the plastic strain perpendicular to crack flank, $\Delta\varepsilon_{p,yy}$, and the crack opening displacement, COD, versus $\Delta K$. These predictions were obtained without contact of crack flanks. The results for different load cases give well defined curves, which clearly point to the validity of the LEFM. Both the plastic strain range and COD increase linearly with the square of $\Delta K$, i.e., $\Delta\varepsilon_{pl}$, COD $\propto \Delta K^2$. Note that the numerical COD was obtained at maximum load, therefore it includes elastic and plastic components. The magnitude of the values found here is according to Pippan and Grosinger [12] who said that in the mid and upper Paris regime the cyclic crack tip opening displacements are in the order of micrometers.

### *Validity of crack closure concept*

Fig. 3 plots the plastic strain range ahead of crack tip versus the stress intensity factor range. Without contact, there is a well defined trend between energy and $\Delta K$. However, there is a great scatter when the energy obtained with contact is plotted versus $\Delta K$. The controversy about the effect of contact has therefore a clear answer: the contact has a significant effect on non-linear crack tip parameters and therefore on fatigue crack growth rate. Nevertheless, when the $\Delta\varepsilon_p$ with contact is plotted versus effective $\Delta K$ ($\Delta K_{eff}$), a well defined trend is obtained once again. Moreover, there is a coincidence of the curves energy without contact versus $\Delta K$ and energy with contact versus $\Delta K_{eff}$. The other two crack tip parameters showed similar results. This coincidence of results clearly shows that the concept of $\Delta K_{eff}$ is able to explain the variations of crack tip parameters produced by the contact of crack flanks.

The results of crack tip parameters versus $\Delta K$ may be seen as master curves, free of the influence of crack closure. Additionally, the results show that without contact of crack flanks there is no effect of stress ratio. Klingbeil [44] also observed that without crack closure, the stress ratio has a negligible effect on total energy dissipation per cycle. Sunder *et al.* [51] observed no effect of stress ratio on crack growth rate of long cracks in 2014-T6511 aluminium alloy (R=0.64; 0.69 and 0.73).

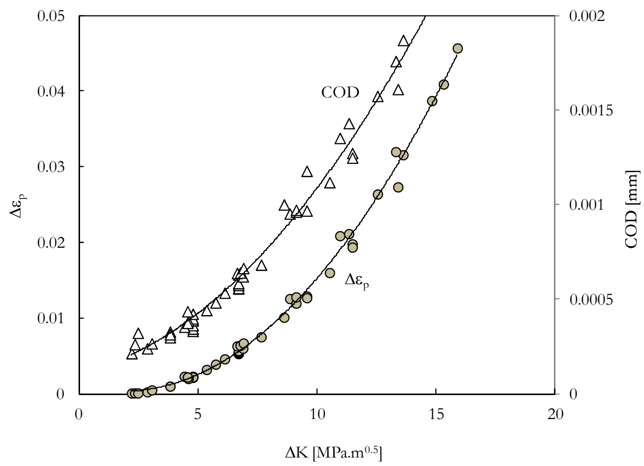


Figure 2: Effect of ΔK on range of plastic strain and crack opening displacement (no contact).

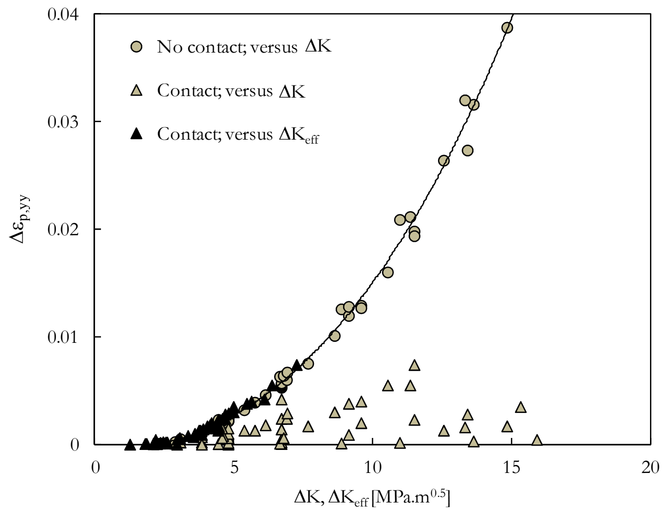


Figure 3: Effect of effective stress intensity factor range, $\Delta K_{eff}$, on plastic strain range.

## *Effect of mesh size*

The finite element mesh is a major parameter of numerical crack closure models. Therefore, the influence of mesh refinement on non-linear crack tip parameters was studied here. Fig. 4 shows the effect of using square elements with 16 $\mu m^2$ at the crack tip (mesh M16), instead of 8 $\mu m^2$ elements (mesh M8), on the plastic strain range. There is a significant decrease of $\Delta\varepsilon_p$, which is explained by the position of the Gauss point relatively to the crack tip. In fact, the decrease of mesh size approaches the Gauss point to the crack tip, as is illustrated in Fig. 4. Anyway, a well defined trend still was observed for mesh M16, which confirms once again the validity of LEFM. The plastic strain range also showed a significant decrease with the increase of mesh size. On the other hand, the COD increased with mesh size. This is explained by the position of the node behind crack tip where the COD was measured. The increase of mesh size departs the node from the crack tip, which increases the COD.

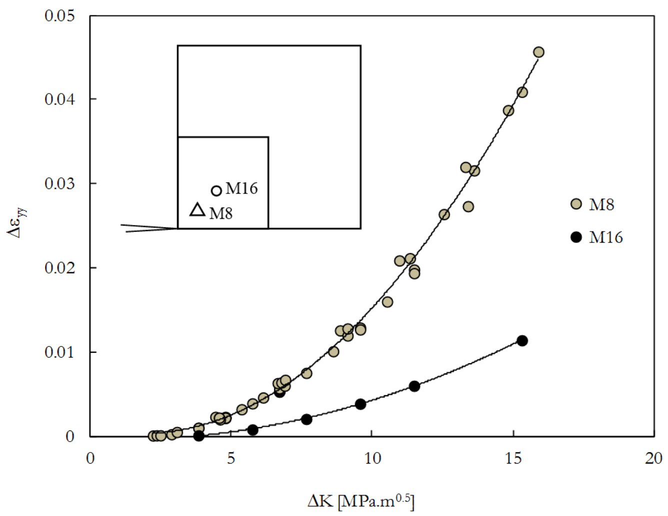


Figure 4: Effect of mesh on plastic strain range.

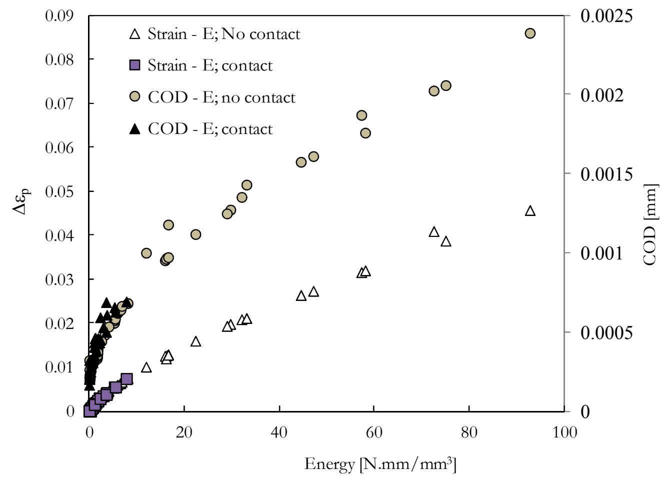


Figure 5: Crack opening displacement versus energy and plastic strain range versus energy.

## *Relations between non-linear crack tip parameters*

Robust relations between the non-linear crack tip parameters were found, with a relatively low influence of mesh refinement. Fig. 5 shows the relations between the specific energy dissipated immediately ahead of crack tip (E) and the crack opening displacement, and between the energy and the plastic strain range. As can be seen, the contact of crack flanks does not affect the relation between the crack tip parameters, which indicates that these are quite robust

*Effect of overloads*

After an overload the crack growth rate has usually a sudden increase followed by a progressive decrease down to a minimum value, and a progressive increase to the pre-overload FCGR. Several mechanisms have been proposed to explain crack growth retardation, namely residual stresses [52], crack closure [3], crack tip blunting, strain hardening [53], crack branching [54] and reversed yielding [55].

The non-linear crack tip parameters were considered here to analyze the effectiveness of crack closure concept to explain the variations of fatigue crack growth after an overload. Several overloads were considered, which are indicated in Tab. 2. Fig. 6 plots the results obtained for the maximum energy ahead of crack tip. The master curve was obtained without contact of crack flanks. When the energy is plotted versus $\Delta K$ there is a great scatter. However, the consideration of $\Delta K_{eff}$ moves the points towards the master curves. This indicates that the crack closure concept is able to explain the variations of non-linear crack tip parameters after an overload, and therefore the variations of da/dN. There is a slight difference between some of the energy-$\Delta K_{eff}$ points and the master curve, which may be explained by partial closure. Further work is required to clarify this issue.

| $\sigma_{min,BL}$ | $\sigma_{max,BL}$ | $\sigma_{max,OL}$ |
|---|---|---|
| 20 | 46.7 | 60 |
| -6.7 | 46.7 | 60 |
| -33.3 | 46.7 | 60 |
| -46.7 | 46.7 | 60 |

Table 2: Overload parameters ([$\sigma_{min,BL}$, $\sigma_{max,BL}$, $\sigma_{max,OL}$,]=MPa)

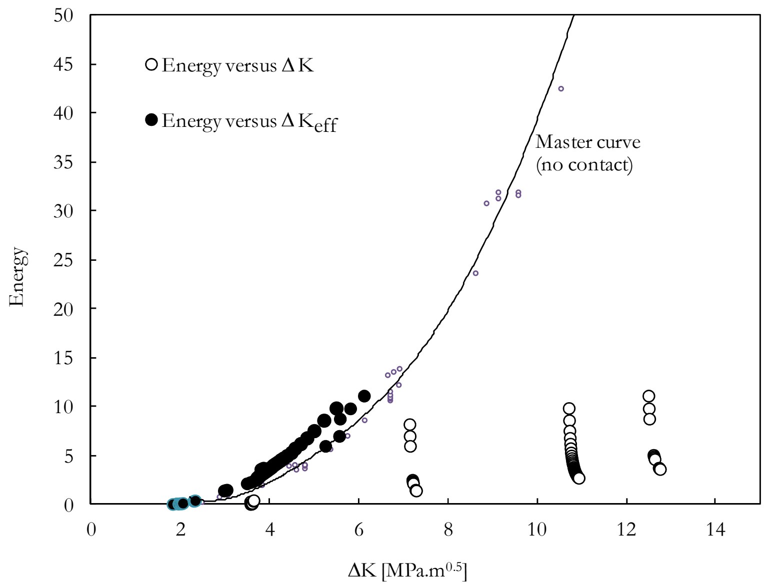


Figure 6: Effect of effective stress intensity factor range, $\Delta K_{eff}$, on dissipated energy after overloads.

## CONCLUSIONS

A numerical study was developed to understand the effect of crack flank contact on crack tip parameters and therefore on da/dN. One of the main outcomes of this work is that the crack closure concept is valid, at least for plane stress state. Additional conclusions resulting from this investigation are:

- the stress intensity factor was found to control the non-linear crack tip parameters (COD, $\Delta\varepsilon_p$, $r_{pc}$, energy). Therefore the LEFM concepts are applicable to the problem being studied.
- the contact of crack flanks, i.e. the crack closure, has a great influence on crack tip parameters that are supposed to control fatigue crack growth rate. The contact decreases the values of the different crack tip parameters.
- the $\Delta K_{eff}$ concept is able to explain the variations of crack tip parameters produced by the contact of crack flanks. The crack tip parameters versus $\Delta K$ obtained without contact may be seen as master curves. Additionally, without contact of crack flanks there is no effect of stress ratio, and $\Delta K$ is the controlling parameter.
- the change of mesh size modifies the relations between non-linear crack tip parameters and $\Delta K$, however the validity of LEFM and of $\Delta K_{eff}$ concept still are verified. The relations between non-linear crack tip parameters were found to be independent of mesh size.
- the crack closure concept is able to explain the variations of non-linear parameters (and therefore of da/dN) after an overload.

The crack tip parameters proved to be a fundamental tool to understand the crack closure phenomenon. In fact, they supply a link between crack closure and fatigue crack growth rate. A close look to non-linear crack tip parameters, like plastic strain range or dissipated energy, is the key for a deeper understanding of FCG and the establishment of physically based relations with loading parameters.

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